

Practice for Final Schneider  
Always give reasons

1. (15 points) Write down the number of possibilities in the following problems (you can leave in symbolic form with numbers in correct places)
  - a. The number of different letter arrangements from **tessellate**
  - b. How many different 7 card poker hands are there (you get dealt 7 cards to look at)
  - c. How many different binary numbers are there of length less than or equal to 10.
  - d. You have 16 people in a class. How many ways can they line up for lunch?
  - e. How many outcomes of 20 flips of a coin have less than 5 heads?
2. (20 points) Fifty five percent of the students at a certain school wear neither a ring nor a necklace. Twenty percent wear a ring and forty percent wear a necklace. If one of the students is chosen randomly what is the probability that the student is wearing.
  - a. a ring or a necklace?
  - b. a ring and a necklace?
3. (15 points) Suppose for the two events A, B we know  $P(A|B)=.2$ ;  $P(B)=.5$ ; and  $P(AB^c)=.2$ . Find  $P(B|A)$ .
4. (30 points) Suppose that a fair die is independently rolled twice.
  - a. What are the probability mass functions for the random variables  $R_1, R_2$  that are the values of the first and second roll respectively? (Graph them)
  - b. What is the cumulative density function of  $R_1$ ? Graph it.
  - c. Graph the probability mass function of the random variable  $3 \cdot R_1$ .
  - d. Graph the probability mass function of the random variable  $R_1 - 3$
  - e. What is  $E(R_1)$  and  $E(R_2)$ ?
  - f. What is  $\text{Var}(R_1)$  and  $\text{Var}(R_2)$ ?
  - g. Consider the random variable  $S=R_1+R_2$  (the sum of the rolls) What is  $P(\{S=6\})$ ?
  - h. Consider the random variable  $M=R_1 \cdot R_2$  (the product of the rolls). What is  $P(\{M=6\})$ ?
  - i. What is  $E(S)$ ?
  - j. What is  $E(M)$ ?
  - k. What is  $\text{Var}(S)$ ?
5. (30 points) Use theorems or definitions to show:
  - a.  $\text{Var}(aX) = a^2(\text{Var}(X))$  where  $a$  is some number. (do not use this fact to prove itself)
  - b. Give an example where  $\text{Var}(X_1+X_2) \neq \text{Var}(X_1)+\text{Var}(X_2)$
  - c. Let  $X_1, X_2, \dots, X_n$  be independent random variables having a Bernoulli distribution ( $P(\{X_i=1\})=p$ ;  $P(\{X_i=0\})=q$ ):
    - i. what is the  $E(X_i)$ ?
    - ii. what is the  $\text{Var}(X_i)$ ?
    - iii. If  $A= (X_1+X_2+\dots+X_n)/n$  what is the  $E(A)$ ? ( $A$  is the average)
    - iv. What is the  $\text{Var}(A)$ ?